IMPELLER FATIGUE ASSESSMENT
USING AN S-N APPROACH

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Abstract

This report details the application of an S-N method to predict the fatigue life in welded joints. It illustrates some of the challenges that engineers will find when using Finite Element methods to determine weld fatigue life. A proposal is made for modifications to centrifugal fan fatigue life specifications for large industrial and power plant fans. This proposal includes prediction of weld life using FEA and S-N methods.

Introduction

During the industrial revolution, at the early part of the 19th century, a series of failures occurred in bridges, chains, pressure vessels and axles. Examination of these indicated that metal could fail when subjected to stresses below yield strength. Figure 1 illustrates the result of failure of the Tay Rail Bridge in 1879.
It was recognised that the repeated application of a load could reduce the strength of metal structures. These observations led to the challenge presented to engineers of describing and predicting fatigue.

Work on understanding metal fatigue started in 1829. By 1837 Albert had completed and published the results of the first fatigue tests. In the period between 1852 and 1870 Albert’s initial work was expanded by Fairbairn, Rankine and Wohler. They included the results of fundamental experimentation into a method used to help predict fatigue life. Wohler could justifiably be called the father of fatigue analysis. Some of his railway axle experiments ran for years. After this date the work on fatigue expanded throughout the whole engineering community.

Some of the highlights of this work are as follows.

<table>
<thead>
<tr>
<th>Date</th>
<th>Originator</th>
<th>Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>1837</td>
<td>Albert</td>
<td>First S-N experiments</td>
</tr>
<tr>
<td>1858</td>
<td>Wohler</td>
<td>Railway axle work, proposal for infinite life, larger safety factor requirement, Wohler curve produced</td>
</tr>
<tr>
<td>1889</td>
<td>Kirch</td>
<td>Stress concentration effects identified</td>
</tr>
<tr>
<td>1920</td>
<td>Griffith</td>
<td>Fracture mechanics foundation</td>
</tr>
<tr>
<td>1924</td>
<td>Palmgren</td>
<td>Damage accumulation principle</td>
</tr>
<tr>
<td></td>
<td>Serensen</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Miner</td>
<td></td>
</tr>
<tr>
<td>1943</td>
<td>Corant</td>
<td>Finite element method foundation</td>
</tr>
<tr>
<td>1950</td>
<td>Palgren &amp; Miner</td>
<td>Reliability requirement defined</td>
</tr>
<tr>
<td>1954</td>
<td>Manson &amp; Coffin</td>
<td>Low cycle fatigue method</td>
</tr>
<tr>
<td>1958</td>
<td>Irwin</td>
<td>Linear fracture mechanics foundation</td>
</tr>
<tr>
<td>1958</td>
<td>Gurney</td>
<td>Welded bridge design standard BS153</td>
</tr>
<tr>
<td>1960</td>
<td>Maddox</td>
<td>Weld crack initiation and crack growth study</td>
</tr>
<tr>
<td>1962</td>
<td>Paris</td>
<td>Crack growth rate equation</td>
</tr>
<tr>
<td>1963</td>
<td>NASTRAN</td>
<td>Commercial FEA available</td>
</tr>
<tr>
<td>1970</td>
<td>Maddox &amp; Gurney</td>
<td>Revised bridge design standard BS5400</td>
</tr>
<tr>
<td>1993</td>
<td>Maddox</td>
<td>Fatigue of welded steel design standard BS7608</td>
</tr>
<tr>
<td>1995</td>
<td>International Institute of Welding</td>
<td>Weld hot spot stress determination method published</td>
</tr>
</tbody>
</table>

The work on understanding and combating fatigue is still active.
After nearly two hundred years of research and practical work, 90% of all failures are still caused by fatigue. Figure 2 illustrates the consequences of a recent ship fatigue failure.

![Figure 2 Tanker fatigue failure](image)

This report gives a brief summary of three basic methods of predicting weld fatigue life. The report then examines in detail the S-N method of weld fatigue life prediction and finally gives details on future developments.

**Weld Fatigue Life Prediction Methods**

**S-N Method**

This method is derived from data generated from physical tests on actual welded test specimens. Results of these tests were used to produce a series of S-N curves for each weld configuration and loading type. For each weld configuration this data is then reduced to a single equation which relates the applied nominal stress range, $S_r$, and the number of cycles to failure, $N$.

The equation form is: 

$$ N = \frac{C}{S_r^m} $$

Where,  

$C = $ weld type design constant  
$m = $ inverse slope of the S-N curve  
$S_r = $ applied stress range  
$N = $ number of cycles to failure

By calculating the joint nominal stress range, using a combination of tensile and bending stress without any weld stress concentration effects, the weld life can be estimated.
Low Cycle Strain Method

The strain life method was originally developed to predict the life of non-welded components. By including weld heat affected zone, HAZ, material data, and an estimated weld toe radius this method has been developed to predict welded component fatigue life. The method comprises of the following steps.

First the nominal stress range, $\Delta S$, at the weld must be determined.

Next the local stress range, $\Delta \sigma$, is determined using a fatigue concentration factor which includes notch ductility effects: -

$$K_t = 1 + \frac{(K_t - 1)}{(1 + \frac{a}{r})}$$

where, theoretical concentration factor = $K_t$
notch radius = $r$
materila constant = $a$

Then the local stress range, $\Delta \sigma$ by combining the cyclic stress-strain equation for the material and Neuber’s local stress rule expression: -

$$\frac{(K_t \Delta S)^2}{E} = \frac{\Delta \sigma^2}{E} + 2\Delta \sigma \left(\frac{\Delta \sigma}{2K'}\right)^{\left(n_\gamma\right)}$$

where, cyclic strength coefficient = $K'$
cyclic strain hardening exponent = $n'$
elastic modulus = $E$

Then the local strain amplitude, $\Delta \varepsilon/2$, using the cyclic stress-strain equation: -

$$\frac{\Delta \varepsilon}{2} = \frac{\Delta \sigma}{2E} + \left(\frac{\Delta \sigma}{2K'}\right)^{\left(n_\gamma\right)}$$

Using this fatigue prediction method for welded structures the applied mean stress, $S_m$, must be included in the life prediction. For structures that have been post weld heat treated, the mean stress magnitude, transverse to the weld, is estimated as: -

$$S_m = 0.3 \times \sigma_{0.2HAZ} + \frac{\Delta S}{2}$$

In cases where no heat treatment is applied the mean stress increases to: -

$$S_m = \sigma_{0.2HAZ} + \frac{\Delta S}{2}$$
The fatigue life can now be predicted using the Morrow method, including the mean stress term:

\[
\frac{\Delta \varepsilon}{2} = \left( \frac{(\sigma' - \sigma_m)}{E} \right)(2N)^b + \varepsilon' (2N)^c
\]

Where:

- Fatigue strength coefficient = $\sigma'_f$
- Fatigue strength exponent = $b$
- Fatigue ductility coefficient = $\varepsilon'_f$
- Fatigue ductility exponent = $c$

Solving this equation gives the predicted fatigue life, $N$.

**Fracture Mechanics**

In the fracture mechanics approach, an initial defect size and shape has to be assumed. The rate of growth of this initial crack size is then predicted using the Paris law crack growth equation.

\[
\frac{da}{dn} = A(\Delta K)^m
\]

Where,

- $da/dn$ = crack growth rate
- $\Delta K$ = stress intensity range
- $A$ and $m$ = material and loading constants

The increase in crack size can then be calculated for a known number of cycles. Failure is indicated if the crack grows to such an extent that the stress intensity factor exceeds the fracture toughness of the material. The assessment can be extended to include the likelihood of failure due to plastic collapse.

This is achieved by calculating two ratios:

\[
K_r = \frac{K_i}{K_{mat}} \quad S_r = \frac{\sigma_{ref}}{\sigma_f}
\]

Where:

- $K_i$ = stress intensity factor
- $K_{mat}$ = fracture toughness of the material
- $\sigma_{ref}$ = reference stress
- $\sigma_f$ = flow stress

These $K_r$ and $L_r$ ratios are then plotted on a failure diagram as shown in figure 3.
This method is fully detailed in British Standard BS7910 Guide to Methods for Assessing the Acceptability of Flaws in Metallic Structures.

**S-N Method**

All three methods of predicting fatigue life have distinct advantages and disadvantages. The primary advantages with the S-N method are that it is based on real test data; this data has been published in various international standards and has been extensively used in all types of welded structures.

The first design standard for predicting weld fatigue life using the S-N method was produced in 1958. Since this date a range of international S-N design codes have been release and are in use today, e.g.

<table>
<thead>
<tr>
<th>Standard</th>
<th>Number</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>British Standard</td>
<td>BS7608 1993</td>
<td>Fatigue Design and Assessment of Structures</td>
</tr>
<tr>
<td>American National</td>
<td>ANSI/AWS D1.1</td>
<td>Structural Welding Code</td>
</tr>
<tr>
<td>Norwegian Oil Industry</td>
<td>NORSOK N-004</td>
<td>Design of Steel structures</td>
</tr>
<tr>
<td>European</td>
<td>EUROCODE 3: - ENV 1993-1-1</td>
<td>Design of Steel Structures Fatigue</td>
</tr>
<tr>
<td>Heath and Safety Executive</td>
<td>21.2.10-15 4th edition</td>
<td>Guidance Notes Offshore Installations</td>
</tr>
<tr>
<td>International Standards Organisation</td>
<td>ISO/CD 13819 part 2</td>
<td>Offshore Structures part 2 Fixed Steel Structures</td>
</tr>
<tr>
<td>American Petroleum Institute</td>
<td>API RP2A WSD</td>
<td>Recommended practice Fixed Offshore Platforms</td>
</tr>
<tr>
<td>International Institute of Welding</td>
<td>XIII-1539-96/XV-845-96</td>
<td>Fatigue Design of Welded Joints and Components</td>
</tr>
</tbody>
</table>

All of these standards are based on the same principle. Data from weld test specimens is used to generate an S-N curve. Figure 4 illustrates the S-N curves from BS7608.
These curves are then converted into equations which relate the applied stress range, $S_r$, to the fatigue life, $N$. Examination of equations, for fillet welds, in each of these standards shows that they give similar results.

<table>
<thead>
<tr>
<th>Design code</th>
<th>SN Line Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>EUROCODE 3</td>
<td>$\log N = 11.701 -3 \log S_r$</td>
</tr>
<tr>
<td>IIW</td>
<td>$\log N = 11.699 -3 \log S_r$</td>
</tr>
<tr>
<td>Norsok</td>
<td>$\log N = 11.699 -3 \log S_r$</td>
</tr>
<tr>
<td>HSE</td>
<td>$\log N = 11.633 -3 \log S_r$</td>
</tr>
<tr>
<td>BS7608</td>
<td>$\log N = 11.634 -3 \log S_r$</td>
</tr>
<tr>
<td>ISO</td>
<td>$\log N = 11.630 -3 \log S_r$</td>
</tr>
</tbody>
</table>

Each of the above S-N curves has the same gradient, but a different constant. To illustrate the effect of the constant, the stress range required to give a life of 10,000 fan start/stop cycles has been determined and is shown below.

<table>
<thead>
<tr>
<th>Design code</th>
<th>Applied Stress $S_r$ ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>EUROCODE 3</td>
<td>53.5</td>
</tr>
<tr>
<td>IIW</td>
<td>53.4</td>
</tr>
<tr>
<td>Norsok</td>
<td>52.5</td>
</tr>
<tr>
<td>HSE</td>
<td>50.8</td>
</tr>
<tr>
<td>BS7608</td>
<td>50.8</td>
</tr>
<tr>
<td>ISO</td>
<td>50.6</td>
</tr>
</tbody>
</table>

Since all the standards are fundamentally equivalent, BS7608 will be used to illustrate the method in this report.
Centrifugal Impeller Weld Types

In each of the design standards, a series of figures are provided to illustrate the weld configuration and loading direction. For centrifugal fans the welds of interest are the joints between the blade and centre, back or side plates.

Three basic types of welds are in common use at these locations: fillet, partial penetration or full penetration.

Figure 5 shows the centrifugal fan weld classifications from BS7608, table 8.

<table>
<thead>
<tr>
<th>Weld Configuration</th>
<th>Weld Class</th>
<th>Sketch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full penetration</td>
<td>F</td>
<td>![Sketch of full penetration weld]</td>
</tr>
<tr>
<td>Fillet or partial penetration</td>
<td>F2</td>
<td>![Sketch of fillet or partial penetration weld]</td>
</tr>
</tbody>
</table>

Figure 5 Centrifugal fan weld classes
The design S-N curves for class F and F2 are shown in Figure 6.

![BS7608 F & F2 Sr-N Design Curves](image)

Figure 6 Centrifugal fan S-N curves

These curves reduce to the following design equations:

\[
\text{Class F2} \quad N = \frac{0.43 \times 10^{12}}{Sr^3} \quad \text{Class F} \quad N = \frac{0.63 \times 10^{12}}{Sr^3}
\]

A prediction for weld design life can be obtained by applying the appropriate equation.

**Limits and User Guidance**

There are limits and conditions which must be applied to the calculation procedures. These are:

1) The standard is applicable to wrought steel operating below the material creep temperature.

2) The method is independent of the steel grade used in the construction. This means that the material constants which are required for the low cycle methods are not required. The reason for this is that all new welds will have micro cracks, approximately 0.004 inch deep, in the weld toe area. Figure 7 shows a micro crack example.
These micro cracks are one of the major differences between parent metal, not welded, and welded metal fatigue behaviour. In parent metal the fatigue life is a combination of a crack initiation cycles and crack growth cycles. In welded metal the crack is already present. This effect is shown in figure 8 which illustrates S-N curves for both non-welded and welded metal.
Since the crack growth rate for all welded steel is approximately equal, the method can be applied to all steel types. Figure 9 illustrates the similar crack growth rate for multiple weld materials. This illustration has been obtained from the ASM Handbook Volume 19, page 440.

3) The standard is applicable to materials with yield strengths less than 102 ksi.
4) Due to presence of welding residual stresses the fatigue life is independent of the applied mean stress. Figure 10 shows two examples of typical welding residual stresses.

![Diagram of welding residual stress fields](image)

**Figure 10 Welding residual stress fields**

Figure 11 shows the effect of applying a tensile stress to the residual stress field. Repeated application of $S_{\text{max}}$ generates a stress range of $S_{\text{ys}} - (S_{\text{ys}} - S_{\text{max}})$ which is equal to $S_{\text{max}}$. 
The fatigue strength of welded joints is, therefore, dependent on stress range and independent of the mean stress. This means that a single S-N curve can be used to represent any minimum and maximum stress ratio. Figure 12 illustrates this effect.

The independence of weld fatigue life with respect to mean stress has been confirmed by testing. These tests were carried out on steels with ultimate strengths that ranged from 77ksi to 121ksi. Each material was tested with various minimum and maximum stress magnitudes i.e. tests with different R values.
This work was reported in TWI report 123/1980 “Some Aspects of the Influence of Residual Stresses on the Fatigue Behaviour of Fillet Welded Joints in Steel” by S.J. Maddox, and has confirmed the independence of weld fatigue to mean stress. Figure 13 illustrates the results from these tests.

These results show that for each material at each R value all the endurance data is within a 95% confidence band.

5) Compressive stress ranges can, and will, produce cracking due to the presence of tensile residual welding stresses. The life due to compressive stress magnitudes must, therefore, be assessed. Figure 14 shows this effect.
6) Any benefit from post weld heat treatment, expressed as a reduction in welding residual stresses, can only be used if the residual stresses are known accurately.

7) It is known that the thickness of the components affects the fatigue strength. In BS7608 this effect is accounted for by increasing the applied stress range. to:

\[ S_r' = S_r \left( \frac{\text{thickness}}{16} \right)^{0.25} \]

8) In centrifugal fan welds, local bending stresses are present. The standard is applicable if the bending stress is less than twice material yield strength.

9) The design equation contains a probability factor of 2.3%. For higher probabilities the standard defines a modification to the equations. Figure 15 illustrates 2.3% or minus two standard deviation effect.

10) The standard defines a stress range, \( S_o \), below which an infinite life can be assumed. This value is the stress range at \( 10^7 \) cycles.

11) The standard defines the fatigue benefit that can be obtained from weld improvement techniques of the weld toe areas;
Determination of the Applied Stress Range

The S-N curves have been generated from results of strain gauge tests and the applied stress range must be determined at the appropriate gauge location. This gauge location is remote from the stress concentration effect of the weld shape and weld toe radius.

Hence the design stress ranges (nominal stress range) are generally determined by classical methods i.e. combination of direct and bending stresses excluding local stress concentration effects. Figure 18 illustrates the required nominal stress.

Substituting the calculated nominal stress range, into the appropriate S-N equation gives the fatigue life prediction.

This method is clearly defined in the standards and has been applied for more than forty years.
Finite Element Method, FEA

For simple structures, determining the stress range is a practical procedure. Centrifugal impellers are not simple structures. The weld fatigue life is dependent on the stress range cubed. This means that classical stress determining methods are not suitable for accurate impeller life predictions.

For complex structures and stress accuracy the best solution is a finite element model of the impeller, Figure 19. This introduces a new challenge.

How do we determine the nominal stress from a finite element model which determines local stress magnitudes? Figure 20 shows the local stress on the blade next to the back plate for the model shown in figure 19.
Three procedures are available to account for local stress magnitudes given by FEA.

1) Set the stress range, $S_r$, equal to the maximum weld toe local stress found in the FEA model. Since this stress will contain the joint geometry stress concentration effect, but no weld toe concentration effect, the resultant stress will be high in comparison to the nominal stress demanded by the SN method. This is a simple procedure which will give a conservative life.

2) Remove the weld joint geometry stress concentration effect by applying a ‘hot spot’ method. In applying a ‘hot spot’ method, the stress magnitudes are lifted from the model at two or three points at pre-determined distances from the weld toe. These stress magnitudes are then extrapolated by either a linear or quadratic curve back to the weld toe location. The resultant extrapolated weld toe stress is then used in the S-N method to predict the fatigue life. Figure 21 illustrates the extrapolation technique.

Figure 21 Stress extrapolation technique

Figure 22 shows the extrapolation methods as defined by International Institution of Welding, IIW. The American Bureau of Shipping, ABS gives a similar method. These two methods produce similar weld toe stress magnitudes.
From the IIW publication 1539-96/XV-845-96 the following extrapolation equation are obtained.

For joints with low bending stress, the IIW two point extrapolation equation is defined: 

$$\sigma_{ns} = 1.67\sigma(0.4t) - 0.67\sigma(1.0t)$$

For joints with high bending stress, the IIW three point extrapolation equation is defined: 

$$\sigma_{ns} = 2.52\sigma(0.4t) - 2.24\sigma(0.9t) + 0.72\sigma(1.4t)$$

In centrifugal fans, high blade bending stresses are present, and the ‘three point’ extrapolation equation is applicable.

In this case, the extrapolated stress will not include the full joint stress concentration effect. The extrapolated stress will be closer to the nominal value required by the S-N method, but will be larger than the nominal stress for the gauge location. The resultant fatigue life prediction will still be conservative.

3) In the extrapolation method detailed above, the stress at the weld toe is estimated. This value is then applied to the S-N curve that has been generated at the strain gauge point. An improvement in the accuracy of the life prediction could be obtained if an S-N curve for the weld toe was available. At present no design standard fulfils this requirement. A recommendation has been published by The Welding Institute, TWI.
In this report the IIW SN FAT90 curve given in the publication 1539-96/XV-845-96, coupled with the IIW extrapolation method is recommended. A comparison of the S-N curves is shown in Figure 23.

![WELD FATIGUE CODES](image)

**Figure 23 S-N curves**

This figure shows that there is a large factor of safety built into all the design standard curves.

**Example**

An impeller FEA has been used as an example. Applying the three methods gives the following results.

<table>
<thead>
<tr>
<th>Stress Method</th>
<th>FEA Sr</th>
<th>IIW Sr</th>
<th>IIW Sr</th>
</tr>
</thead>
<tbody>
<tr>
<td>SN Method</td>
<td>BS7608</td>
<td>BS7608</td>
<td>FAT90</td>
</tr>
<tr>
<td>3 point</td>
<td>50.5</td>
<td>45.0</td>
<td>45.0</td>
</tr>
<tr>
<td>N</td>
<td>10200</td>
<td>14400</td>
<td>49000</td>
</tr>
</tbody>
</table>

These results illustrate the very large conservatism that will be generated by applying FEA directly to the design standard S-N curves. It also shows that the IIW extrapolation method is also potentially conservative.

**Future**

This example illustrates the importance of using a stress range and S-N curve which are based on the same location and use the same stress definition. Research continues around the world on the methods used to extract the correct weld toe stress from FEA models and application of this to an appropriate S-N curve. A summary of two of these methods is given below.
**Nodal Force Method**

In this method the nodal forces are extracted from each node in the FEA model. These forces are then used to generate membrane and bending loads along the weld toe line. By applying an energy balance technique, it is possible to generate an equivalent structural stress at the weld toe line. This can then be combined with the master S-N curve to produce a fatigue life prediction.

This method is now commercially available from Battelle via the VERITY program.

In 2007 the VERITY program was incorporated into the pressure vessel design code ASME 8.

**TWI Research Project**

The welding Institute, TWI, are at present running a major international project which has the following objectives.

1) Provide a unified and coherent framework of the three levels of assessing the fatigue life of welded joints, using the nominal stress, structural stress and equivalent stress methods.
2) Validate and recommend procedures for FEA-based fatigue design.
3) Reduce design costs, and improve reliability and safety.
4) To encourage adoption of the new procedures by all Standards bodies.

This project started in 2006 and is due for completion in 2008.

The project is being supported by a range of industrial users (including Lloyds Register, Howden Technology, Jaguar Land Rover, Dalmine, TATA, Bombardier), government bodies (including the UK department of Trade and Industry) and universities (including Strathclyde and Glasgow).

The project has examined three methods of extracting the appropriate stress range for inclusion in a single S-N curve for all welds.

From these examinations, preliminary conclusions on the FEA mesh sensitivity and accuracy have been obtained.

**Discussion**

It is general practice in the USA for large fan contract specifications to include a demand for the low cycle fatigue prediction method of Fatigue Life. In this report an alternative method has been described. Figure 24 shows the consequences of changing the specification to this method.
This illustrates that the low cycle method underestimates the fatigue life in the start stop cycle area of the curve.

Conclusions

To determine the fatigue life of complex welded structures the following procedure is recommended.

1) The SN method is preferred, due to its simplicity and the fact that is based on real test data.

2) FEA techniques must be applied to generate accurate stress ranges. This is considered appropriate due to the complexity of the stress field within centrifugal impellers.

3) Determine the fatigue life by applying the local stress range from the FEA model into the appropriate SN curve equation.

4) If the fatigue life determine by 3) is insufficient, use the IIW stress extrapolation method.

5) The specification of this procedure should be reviewed when the FEA generated fatigue life research work is completed.

6) Conclusions of the fatigue research work should be included in the international fan design standards.