ANALYTICAL MODEL FOR AXIAL FAN PERFORMANCE RATING

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SUMMARY
Current requirements of AMCA 211 only allow interpolation on one geometric variable (such as blade pitch, hub to tip ratio, number of blades) for geometrically similar fans, and the results have to be verified by tests. A similar allowance is made for solidity. This standard results in massive testing requirements, especially for axial fans with adjustable pitch blades. The paper investigates methods to reduce the testing requirements where the sizes are not geometrically similar and multiple geometric variables are changed.

An analytical model of axial fan performance was developed, validated, and applied towards rating of 3 product lines. The wheel diameter was varied from 12.15 to 78.74 inches, hub ratios from .19 to .56, blade angle from 10° to 40°, blade count from 2 to 12, and solidity from .09 to 1.27. None of the various sizes were geometrically similar.

Full rating procedure per AMCA 211-05 CRP would require a total of 129 air tests with varying fan size, blade count and hub ratio (903 including blade angle variation). Interpolation on one variable per AMCA 211-05 would save about 50% of laboratory testing. Model application reduced the amount of testing by at least 80% while keeping the rating pressure and power data within AMCA 211 tolerances 90% of the time for solidity below 0.65. The model basically employs aerodynamic principles tested and validated across a multi-variable parameter space.

It is the authors’ hope that AMCA will consider this approach as acceptable in its future CRP standards for air performance ratings. It is also the authors’ opinion that AMCA should administer the CRP based on performance ratings furnished by the manufacturer while being open to various innovative methods of obtaining these ratings.

HISTORICAL OVERVIEW OF AMCA 211 CRP STANDARD REGARDING CALCULATED AIR PERFORMANCE OF AXIAL FANS:
The first edition of AMCA 211-1965 states:

C5.3 The performance of a series of Axial Fans that are identical except for one variable (such as blade angle, number of blades, hub to tip diameter ratio, etc.) may be derived by interpolation between tested fans, provided that a clear relationship of the variable to the fan performance can be demonstrated. Calculation to larger sizes may be based on interpolated performance provided that the requirements of proportionality are met. It is incumbent upon the manufacturer to establish the validity of the relationship and to insure that the interpolation method used is consistent and results in rating within the Performance Tolerances.

In the same document, AMCA specifies what values should be used as ‘performance data’.

C7.1 Ratings Not Related to Particular Motor Sizes
CFM
SP and/or TP
BHP
Impeller RPM at standard or specified inlet density
In 1974, words “…within the Performance Tolerances” were changed to “…within the Check Test Tolerances”, and in 1994 to “…within the Certified Ratings Program Tolerances.” No other significant changes were made to the standard regarding interpolated and/or calculated axial fan performance ratings at that time.

In 1998, paragraph C5.3 appears in the standard as D4.2.1, and next to it follows a new paragraph that allows establishing relationship between certain variable(s) and fan performance.

**D4.2.2 SOLIDITY**

The performance of an axial fan product line that is not geometrically identical, but is geometrically alike, can also be derived for units where an important dimension and/or attribute have been altered. This would include changing blade numbers and chord while maintaining blade similarity (geometric similarity and the same airfoil sections used) and blade solidity (ratio of total of blade chords divided by swept circumference). In such cases it may be possible to derive the aerodynamic performance (but not the sound) of the fan range or parts thereof, by testing a number of units (a minimum of three sizes) and demonstrating that a clear identity exists between the fan performance and the variable(s).

The procedure for establishing the relationship is the responsibility of the manufacturer, but it must be based on accepted theory supported by data from experimental tests conducted to the same standard and test methods as that used for the remainder of the range.

In 2005, this paragraph was split into two, and words “accepted theory” were removed from the second part

**12.1.2.3 Establishing relationship**

The procedure for establishing the relationship is the responsibility of the manufacturer, but it must be based on data derived from tests conducted to the same standard and test methods as that used for the remainder of the range.

The rating method proposed in the present paper is based upon airfoil theory and boundary layer theory. Therefore, AMCA 211-94 rev 11/98 would be the best starting point for a possible revision of the current CRP standard. On the other hand, model parameters are “based on data derived from tests”, and hence AMCA 211-05 would also apply.

![Fig 1. Tube axial Fan Setup](image)
ASSUMPTIONS AND CONCEPTS:

The present model predicts air performance of a typical tubeaxial fan whose setup is shown in Fig 1. The model assumes inviscid, low turbulence flow away from solid surfaces and fully-developed turbulent boundary layers close to the blade and tube surfaces, where viscous losses are most likely to occur. The model assumes that thin airfoil blades introduce minimal obstruction to the flow. Losses due to premature separation of boundary layers (e.g. near stall region) as well as very strong solidity effects are modeled separately. Under certain simplified assumptions, all airfoil geometry and dynamics are estimated at the gyration radius of the annular area (not at the blade tip) and taken as representative for the entire blade span. This gyration radius, shown as a pink dashed circle in Fig 1, depends on fan diameter $FD$, hub-to-tip ratio $HR$, and blade shape.

Shaft thrust and torque values are derived from aerodynamic forces (lift and drag) exerted by air on all the fan blades. In the present model, we assume that lift and drag are concentrated within a thin cylindrical ring in the vicinity of gyration radius. Unbending/unrolling this ring gives an infinite cascade of airfoils, as shown in Fig 2. In this figure, the absolute air velocities are represented as solid black vectors $\mathbf{V}$, the air velocities relative to the blade – as solid blue vectors $\mathbf{W}$, and the blade velocity itself – as a green vector $\mathbf{U}$. The aerodynamic force acting on the blade is decomposed into two components: drag (orange) and lift (dark blue). Drag is parallel, and lift is normal to the relative air velocity $\mathbf{W_1}$ at the leading edge. Then, for proper modeling and scaling of fan flow, pressure, and power, we have to analyze angles among all these vectors.

FLOW, POWER AND PRESSURE SCALING

In test lab and production environments, the blade angle $BA$ between the blade chord (shown as a green dashed line) and the plane of fan rotation is typically measured at a fixed distance from the hub. To analyze, predict and catalog performance of various airfoils, NACA documented angle between blade chord and relative air velocity far away from the airfoil and defined such angle as an angle of attack $AA$ (see Jacobs and Anderson 1931). An airfoil cascade, however, not only accelerates but also redirects the air stream (Lojtzyanskij 1973), as one can see in the bottom part of Fig 2. In other words, a rotating axial fan impeller generates a swirl. To analyze the effect of this swirl on pressure and power, it is also important to know the angle at which air separates from the trailing edge, $TA$. For proper modeling of this angle we have to relate it to the flow rate $CFM$ in a non-dimensional form and to apply a classical Euler equation for fans, found in many texts (e.g. Bleier 1997, Wright 1999). To estimate the brake horsepower $BHP$, we need to make some additional assumptions. These
assumptions were verified with experimental data to achieve the best possible fit, as described in the next section.

1) Additional viscous losses must be introduced as a factor to the power scale (Monin and Yaglom 1992).
2) Solidity is defined in the present model as a ratio of the total area occupied by all the blades at $BA = 0$ (see grey blades in the left part of Fig 1) to the total annular area (green circle in Fig 1). The brake horsepower includes solidity into a multiplier deducted from plots in Wright (1999).
3) The unknown trailing edge separation angle is modeled to achieve the best data collapse for all the fan sizes tested so far in the present rating program.
4) Additional terms are added to the power equation to predict fan performance in less efficient flow regimes, such as stall.

Implementation of these assumptions will somewhat modify traditional Euler equation for fans. The new power model will introduce seven unknown parameters:

- $\sigma_0$ – ‘e-solidity’ that is used in the solidity multiplier;
- $F_v$ – ‘viscosity factor’, which models viscous losses on all solid surfaces;
- $F_D$ – drag factor, which quantifies power increase at small angles of attack due to drag;
- $F_{BA}$ – blade angle factor; quantifies the effect of blade angle on drag;
- $\% WOV_S$ – non-dimensional ‘stall flow rate’, or an approximate percent wide-open volume below which stall usually occurs;
- $F_S$ – stall factor and
- $C_S$ – stall constant.

The last three parameters serve to predict stalling regime at least approximately. Such predictions are necessary to avoid rating fans in the stall region.

Fig 3. Balance of Forces on a Blade.

Now, as we know the non-dimensional flow rate and the horsepower, we can calculate the total pressure coefficient from the balance of forces acting on the blades (see Fig 3). Note that in the direction of fan rotation, both lift and drag positively contribute to shaft torque, whereas in the flow direction, drag gives negative contribution to thrust. Simple vector analysis expresses total pressure coefficient $\psi_t$ and total efficiency $\eta_t$ in terms of lift $c_L$ and drag $c_D$ coefficients of the blade airfoil. Thus, to predict the pressure, we need to find a model for aerodynamic performance of the blade airfoil. Drag coefficient $c_D$ can be computed from blade Reynolds number using boundary layer theory and correlations (Monin and Yaglom 1992), and lift coefficient $c_L$ can be approximately modeled based on NACA charts in Jacobs and Anderson 1931. In the model expression for $c_L$, we have to adjust experimentally three additional lift-and-drag airfoil parameters:

- $c_{max}$ is a measure of maximum lift-to-drag ratio;
- $AA_0$ is ‘zero lift angle of attack’ at which zero lift takes place (with an opposite sign).
- $AA_{max}$ is ‘maximum lift angle of attack’.

Altogether, these ten parameters along with a set of equations represent a complete analytical model for predicting air performance of a tubeaxial fan.
Fig 4. Brake Horsepower fit, restricted to peak efficiency.
Fig 5. Brake Horsepower fit, expanded limits.
LIMITATIONS, PARAMETER SPACE, AND ADJUSTMENTS:

During the early stages of model development process, we set very tight restrictions on model variables, such as angle of attack, hub ratio and solidity. Model coefficients were adjusted to achieve the best fit between predicted and measured horsepower values within 0.01 of power scale (which translates into 2.5% of peak horsepower), as shown in Fig 4. In this and the two subsequent figures, $CFM^*$, $TP^*$ and $BHP^*$ stand for flow, pressure and power coefficients, respectively ($\phi$, $\psi$ and $\lambda$). However, in these early stages, the tight restrictions on the angle of attack severely limited model application to the closest vicinity of static or total efficiency peak. Later on, the limits were modified and somewhat expanded to include more data (see Fig 5).

Limits on the angle of attack are set just to avoid inefficient zero-lift regime and very deep stall. Very low solidity cases were excluded because they consistently failed. The application of very high solidity cases was limited to high flow rates. Practically this restriction excluded all the inefficient cases with very large hub-to-tip ratios and overlapping blades. Minimal flow rate was determined by direct analysis of test data, which effectively excluded the stall region with 10% to 15% safety margin.

To date, tubeaxial fan air performance prediction was tested for two out of three recently developed product lines of fan blades. After applying the above limits, the prediction fits the test horsepower to within 2% of horsepower scale (or within 5% of peak horsepower as allowed by AMCA 211, see Fig 5). With the same set of parameters and restrictions, total pressure fit follows about the same trends as power (see Fig 6).

Solidity and blade count were found to be the two major factors affecting both fan air performance and model accuracy. Therefore, to obtain air performance ratings for the first two out of three product lines, many model parameters were adjusted individually and/or interpolated based on solidity, as shown in Fig 7. In this figure, the big color square diagrams labeled ‘Solidity’, ‘Viscosity Factor’ and ‘Drag Factor’ indicate how each of these parameters was changing with blade count (horizontal axis), and fan and hub diameters (vertical axis). The direction of these axes and the order of hub diameters were set so that solidity increases from left to right and from bottom to top. Stall and airfoil parameters (which are represented by upper-right and middle-right squares) were adjusted only when solidity was above 0.65, and therefore only cases with 9 and 12 blades are displayed in these diagrams. In these cases, many model parameters required significant adjustments due to increased solidity, and the shading of these cells varies from orange to black, depending on the solidity level. However, when solidity was below 0.65, the model predicted fan performance accurately with the same set of parameters.
most of the time. Such cases are highlighted with yellow in the figure. Similarly, other colors represent specific parameter adjustments.

About the same relative amount of testing was performed on a different family of smaller blades that were not geometrically similar with the blades discussed above, and another parameter space chart was build. This chart, which looks similar to Fig 7, is not shown here.

RESULTS:
For each fan size and blade count (represented by a single cell in each diagram in Fig 7), we attempted to rate an axial product line at seven blade angles ranging from 10° to 40°. For only one fan configuration (1400 mm wheel diameter, 12 blades) we actually tested all seven blade angles. This configuration is shown in very large bold font and framed in the figure. For another 8 fan sizes (shown in bold) we tested 3 to 4 blade angles and used this test data for model development and validation. A 1400-mm fan with two blades (bold italic) was tested for model development only but not rated for production. Air performances for the other 43 cases (regular font) were predicted using the model. Effectively, instead of 52 fan configurations, we tested 5½ (10.6%). In some cases, however, we had to double the number of data points along the curve for better resolution.

We performed point-by-point comparison between model and test fan curves for the 9 test configurations, and here we present 4 of them: 630mm with 12 blades (highest solidity case, Fig 8), 800mm with 6 blades (a contract test, Fig 9), 1400mm with 3 blades (lowest rated solidity case, Fig 10), and 1400mm with 12 blades (all blade angles tested, Fig 11). Each of these figures compares values of (a) brake horsepower and (b) total pressure. In these plots, solid black lines represent modeling data, dashed lines – AMCA 211 tolerance margin; small green squares indicate test points that fall within, and large red circles – test points outside of the AMCA 211 margin. Modeled air performance in these figures complies with all the limitations mentioned in the previous section except that in Figs 10 and 11 it does not check for the minimal CFM. Figs 10 and 11 indicate that the model fails to predict the beginning of stall, and hence the CFM limitations were obtained from direct manual comparison of the model data with test cases where stall regions were clearly identified.

We set up a Microsoft Excel workbook to calculate the air performance for all the rated sizes using the analytical model. For cases where test results were not available, interpolation of the model parameters (but not of the actual air performance data!) was applied, based on a single variable – solidity. A worksheet similar to Fig 7 was used for interpolation.

CONCLUSIONS AND RECOMMENDATIONS:
1. Modify AMCA 211 standard to allow product certification using predictive models.
2. Significant reduction in product testing costs is possible to develop performance ratings.
3. Manufacturers can focus on creative product development instead of excessive testing requirements.

REFERENCES
Jacobs, Eastman N; Anderson, Raymond F  “Large-scale aerodynamic characteristics of airfoils as tested in the variable density wind tunnel”, NACA-TR-352, 1931
Lojtzyanskij, LG  “Mekhanika Zhidkosti i Gaza”, Moscow, Nauka 1973
Wright, T  “Fluid Machinery: performance, analysis and design”, CRC Press 1999
Fig 7. Model Parameter Space.
Fig 8. Model Validation: $FD = 630\text{mm}$; 12 blades.
Fig 9. Model Validation: $FD = 800\text{mm}$; 6 blades.
a) FD=1400mm, HR=.25, 3 blades, Brake Horse Power at 1000RPM

![Graph showing Brake Horse Power vs CFM for different angles of attack.]

b) FD=1400mm, HR=.25, 3 blades, Total Pressure at 1000 RPM

![Graph showing Total Pressure vs CFM for different angles of attack.]

Fig 10. Model Validation: FD = 1400mm; 3 blades.
Fig 11. Model Validation: \( FD = 1400\text{mm}; 12 \text{ blades}. \)