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PRESSURE AGAINST FLOWRATE – IS THE SQUARE LAW TRUE?

Dr. W.T.W. (Bill) Cory
Principal
Bill Cory Consultancy

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by

W. T. W. (Bill) Cory
DEng, MSc, CEng, FIMechE, MCIBSE, MIAgrE, FRSH, MIIAV Consultant
Past President AMCA, Chairman ISO/TC117 Fans, Chairman Eurovent WG1 Fans

1. Synopsis

For many years we have lived with the statement that the system resistance is proportional to the flowrate squared i.e. that it is a quadratic relationship. Whilst this is **almost** true for small changes in the flow, its shortcomings are beginning to be realized in an era of inverters and other methods of wide ranging speed control. System air velocities are often in the transitional zone, such that the Reynolds Number effects have to be considered. The wider the range of flows, the bigger the observed discrepancies. This paper gives the theoretical reasons for the breakdown of the 'square law'. It also notes that there is no intrinsic loss coefficient for a given duct fitting.

2. Introduction

In the design of a ductwork system it is the practice to add the resistance of all the elements in the index leg together, to determine the total (or static) pressure loss. Allowing for system effects, the fan must develop this pressure at the design flowrate. The system and fan will then be in harmony.

The resistance of duct fittings and straight ducting are invariably determined from the Guides produced by bodies such as ASHRAE or CIBSE. Both organizations have a similar approach and treat the pressure losses as a function of the local velocity pressure. This function is usually regarded as a constant and thus the loss becomes:

$$p_{Lf} = k_F \times \frac{1}{2} \rho v^2$$

where:

p_{Lf} = pressure loss (Pa)
k_F = constant
ρ = local air density (kg/m^3) (usually taken as standard 1.2)
v = local velocity (m/s)

In the Inch – Pound system of units the loss is:

$$p_{Lf} = k_F \times \rho \left(\frac{v}{1096} \right)^2$$

where:

p_{Lf} = pressure loss ins. w.g.
k_F = constant
ρ = local air density (lb/ft^3) (usually taken as standard 0.075)
v = local velocity (ft/min)

Whilst this may be reasonably true in the normal working range, it is important to know that k_F has a Reynolds Number dependence and that at low Reynolds Numbers k_F can increase enormously, whilst in fully turbulent flow, if ever attained, the value could be less.

There are very few textbooks which even admit this variation. Certainly, the ASHRAE and CIBSE Guides say nothing. The only one of note is Miller's *Internal Flow Systems* which gives a very detailed exposition of the subject and is noteworthy for its comprehensiveness. Idelchik's *Handbook of Hydraulic Resistance* is also recommended for pointing out the dependency of k_F on Reynolds Number.

It might be thought that the topic is somewhat esoteric, but it is suggested that with the increasing use of inverters and other variable flow devices, **it is important to know that at high turn-down ratios, the system resistance curve diverges ever more from the often quoted $p_L \propto Q^2$.** Thus power absorbed is **not** $\propto$ fan speed N^3, even if there were no bearing, transmission and control losses. (see ISO 5801 : 2007 Annex E)

In like manner, the loss in straight ducting is usually quoted as:

$$p_{Ls} = \frac{fL}{m} \cdot \frac{1}{2} \rho v^2 \qquad \text{and } \frac{fL}{m} \text{ is taken to be a constant } k_s$$

where:

L = length of straight duct (m)

m = mean hydraulic depth (m)

= cross sectional area ÷ perimeter

= $\frac{d}{4}$ for circular cross-sections

f = friction factor

Again, as L and m are constants and f is assumed to be constant, the loss is taken to be

$$p_{Ls} = k_s \cdot \frac{1}{2} \rho v^2$$

And thus another problem is created, for *f* is **not** a constant but rather a function of absolute roughness and Reynolds Number.

3. Moody Charts

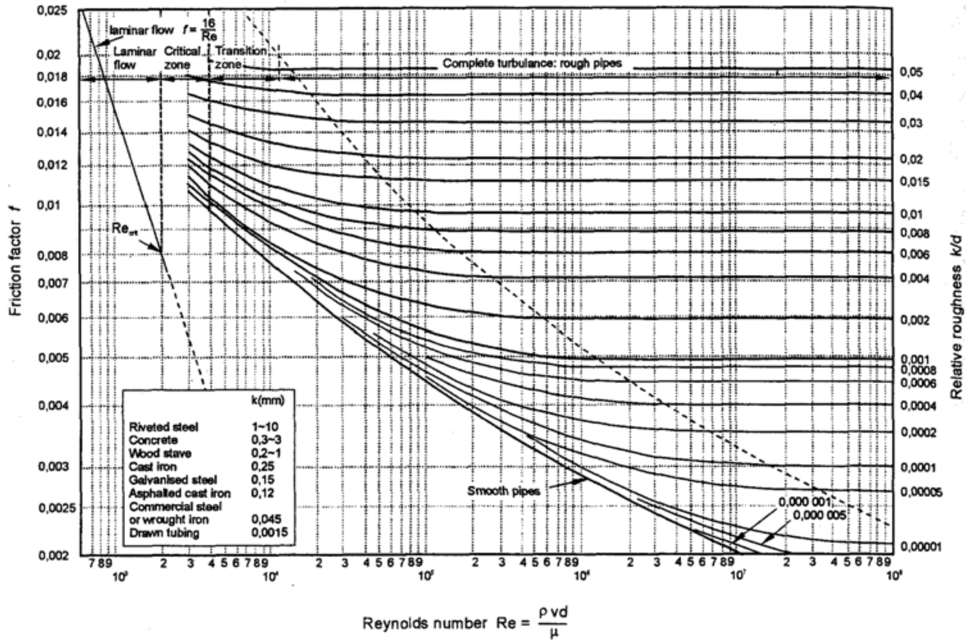

Figure 1a : Friction factor versus Reynolds number – Moody chart (European)

The Moody chart Figure 1a shows that in the transitional and lower zones $f \neq$ constant, and that again, as flow enters the critical zone there are significant increases in *f*, then a sudden drop, before climbing again in the laminar zone.

Here, I have to interject with a warning. Not only are Europe and America divided by units, but also by definitions. In the U.S.A. most textbooks define the friction loss of straight ducting in terms of *d* – its diameter. Thus as $m = d/4$, the values of f are multiplied by 4 (see AMCA 200-95 p9)

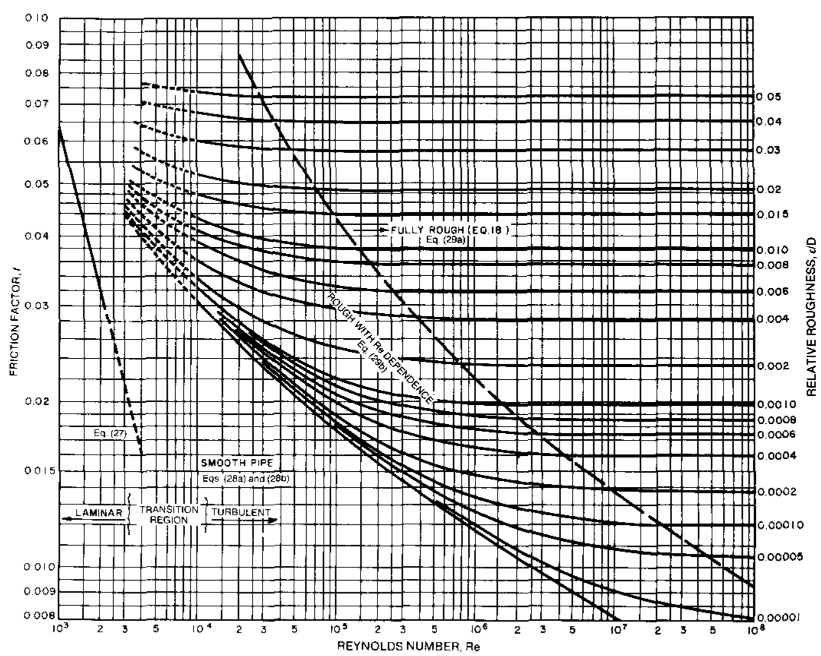

Figure 1b: Friction factor versus Reynolds number – Moody chart (American)

Diameter d m	Average Velocity v m/s	Reynolds No. $Re = \frac{\rho v d}{\mu}$	Relative Roughness $\frac{k}{d}$	Friction Factor f	Flow quality
0.1	2.5	16492	0.0015	0.0076	Tr
	5	32985		0.0067	
	10	65970		0.0063	
	15	98955		0.0059	
	20	131940		0.0057	
0.25	2.5	41231	0.0006	0.006	Tr
	5	82463		0.0055	
	10	164926		0.005	
	15	247388		0.0048	
	20	329851		0.0047	
0.315	5	103903	0.00048	0.0051	Tr
	10	207806		0.0047	
	15	311710		0.0046	
	20	415613		0.0045	
	25	519516		0.0044	
0.63	5	207806	0.00024	0.0043	Tr
	10	415613		0.0042	
	15	623419		0.0039	
	20	831226		0.0038	
	25	1039032		0.0036	
1	5	329851	0.00015	0.0039	Tr
	10	659703		0.0037	
	15	989555		0.0036	
	20	1319406		0.0035	
	25	1649258		0.0034	
2	10	1319406	0.000075	0.0033	Tr
	15	1979109		0.0032	
	20	2638812		0.0031	
	25	3298516		0.003	
	30	3958218		0.00295	
2.5	15	2473887	0.00006	0.00295	Tr
	20	3298516		0.0029	
	25	4123144		0.00285	
	30	4947773		0.0028	
	40	6597031		0.0028	

Table 1: Friction factors versus duct size and velocity (European conventions)

Note 1: Values apply to standard air
Note 2: All values are in the transitional range

Referring now to Table 1, this covers the range of sizes and velocities encountered in HVAC practice. Assuming an absolute roughness applicable to g.s.s. (galvanized sheet steel), it can be seen that in all these cases the flow is transitional. The relative roughness and friction factor therefore vary enormously as shown. Thus with decreasing flow, and therefore velocity, the reducing velocity pressure is partially offset by the increase in f.

Now table 1 has been formulated for metric units, but it would be just the same for inch – pound units. I leave the enthusiastic reader to make a new table for a relevant range of sizes, velocities and roughnesses (with friction factors four times as large!).

4. Laminar Flows

At sufficiently low velocities and small duct sizes the Reynolds Number will be below 2000. The flow becomes laminar away from disturbances. Only viscous forces are of any importance, making shear and hence energy dissipation directly proportional to velocity. In pressure loss terms we may still say that:

$$p_{Lf} = k_F \frac{1}{2} \rho v^2$$

However the loss coefficient k_F for a particular component is then inversely proportional to the local Reynolds Number. Therefore:

$$k_F = \frac{\text{Constant}}{\text{Reynolds Number}}$$

Within duct fittings abrupt changes in area and direction induce turbulence at Reynolds numbers well below 2000. Once turbulence is induced the fitting's loss coefficient is no longer inversely proportional to Reynolds number, nor is it necessarily only mildly dependent on Reynolds number, as it will usually become at Reynolds numbers above 10^4.

Flow patterns can also vary and affect loss coefficients in the purely laminar region. Up to Reynolds numbers of about 10, based upon the maximum velocity within a component, 'creeping' flow without separation is possible at abrupt changes in area. At slightly higher Reynolds numbers inertia forces become important causing laminar separation, followed by laminar re-attachment. A further increase in Reynolds number results in the separated flow becoming turbulent.

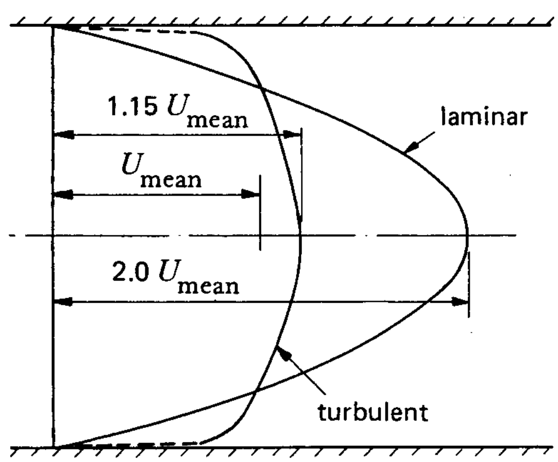

Figure 2: Laminar and turbulent velocity profiles

Referring to Figure 2, developed laminar flow is characterized by a kinetic energy coefficient and a peak to mean velocity ratio of 2, compared to developed turbulent profiles with a kinetic energy coefficient of just over unity and a peak to mean velocity ratio of about 1.2. If, due to turbulence within a component, an initially laminar flow leaves a component with a nearly uniform velocity, energy has to be taken from the mean flow in order to re-establish developed laminar flow. Following a smooth contraction the extra pressure loss over and above the friction loss calculated using developed flow friction coefficients, is 1.3 times the mean velocity pressure. Velocity reduction in laminar flow is accompanied by a greater energy dissipation than the theoretically recoverable velocity pressure, so diffusers have no place when flows are laminar.

5. Transitional Flows

The general shape of loss coefficient versus Reynolds number curves, for transitional flows, is shown in Figure 3. The curves do not apply to smooth bends.

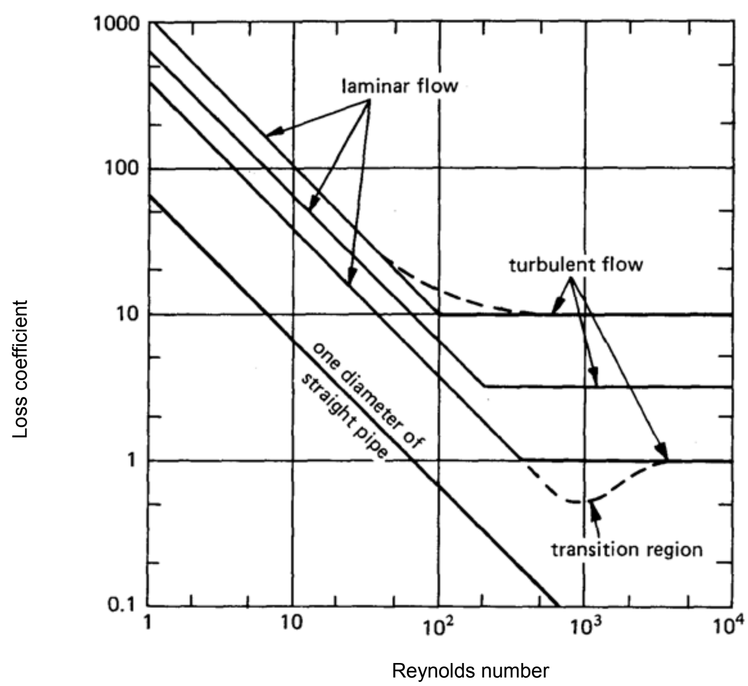

Figure 3: Trends in loss coefficients in the transitional regime

From the above, it is evident that the laminar to turbulent, or transitional regime is the most complex. An example of how complex, is provided by flow through 90° bends. Above Reynolds numbers of 100 inertia forces become important. Centrifugal and static pressure forces acting on the highly peaked laminar velocity profile at inlet to a bend deflect the core region outwards. Secondary flows are set up in a similar manner to those with turbulent flow, but with laminar flow they are stronger and stable. The interaction of the secondary flows with the core region and the effects of flow stability on curvature, tend to delay the onset of turbulence to well above Reynolds numbers at which straight duct flow would become turbulent. At the same time as the onset of turbulence is being suppressed, the secondary flows grow in strength increasing viscous energy dissipation within the bend and in the outlet duct. Because of this suppression, a turbulent inlet flow may become laminar, but highly distorted, and return to turbulent flow in the outlet duct. The complex phenomena that occur in the transition region are reflected in the shape of the bend loss coefficient curve in Figure 4. Also shown in the figure is the approximate loss coefficient for an abrupt bend of centerline radius to diameter ratio of 0.7, which has a lower loss coefficient at

Reynolds numbers of 1000 than larger radius ratio bends. It should be noted that the 'ledge' in the curves is the normal 'constant' given in ASHRAE and CIBSE data.

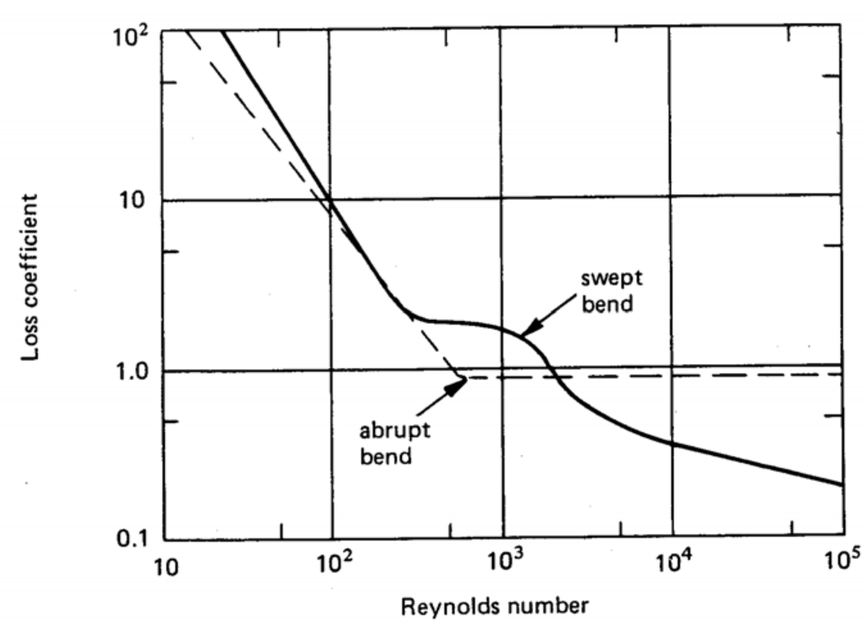

Figure 4 : Bend loss coefficients for transitional flows

6. Interaction Effects

We have observed previously in this document that the total pressure loss for fan selection can be deduced by summating the losses of individual elements in the index leg of our system. This is easier said than done. As stated in Eurovent 2/9, there is no intrinsic value of energy loss coefficient for any component. For each upstream flow condition a different value will be found. The usual convention in any test work has been to use a long straight duct both upstream and downstream of the element such that a fully developed profile, free from swirl, is both presented to, and regained after the element. From these conventions, it will be immediately recognised that the assumption that the pressure loss in a component, assumed as:

$$p_{Lf} = \text{coefficient } x \text{ velocity pressure}$$

can only be correct under very narrow conditions. It is not correct to state that the coefficient is a constant, over a range of duct sizes and air velocities. A very strong Reynolds N^{o} influence may be anticipated, for this will determine the ratio of centre line (peak) velocity to mean $\left(\frac{q_v}{A}\right)$ velocity.

It has been noted that conditions in most air systems are neither fully turbulent nor truly laminar but rather 'transitional'. Aside from frictional considerations, most easily determined by reference to the Moody chart (fig 1), it will be appreciated that in conjunction with size, this will determine the thickness of the boundary layer. As a proportion of the duct diameter or equivalent, the variation may be enormous.

The above is merely an introduction to the vexed question of interaction effects when two bends or similar components are placed close to each other. If the bends are separated by a straight duct of say 30 diameters it may be expected that the loss of the two bends is the simple addition of the individual losses (plus of course the loss for the intervening straight duct). A little consideration of the velocity profiles will show why this is not the case in a typical working environment. Due to inertia forces (centrifugal and static pressure forces) acting on the peaky velocity profile at inlet to a bend, the core region is deflected outwards. Hence the velocity profile to the second bend is far from symmetrical and the loss coefficient for it is no longer valid.

Whilst the few text books and guides, which are aware of the problem, usually state that neglecting interaction effects will frequently result in an over estimation of pressure losses, it should be expected that this is far from the case with tight bends $\left(\frac{r}{d} < 1\right)$ or mitred bends. It should be especially noted that much of the data has been obtained from tests on water in smooth pipes where the flow is invariably fully turbulent. It is highly unlikely that it can be translated to the transitional or laminar flow in air duct systems without significant error.

Finally it should be realised that the juxtaposition of two bends in a duct run can induce swirl downstream. This swirl may not decay for over 100 diameters of straight duct. Consequently the 'wetted' path is greatly increased in length and hence its pressure loss may be much higher than that calculated for the assumed straight line, fully developed swirl free flow.

7. Conclusions

Those of you still hanging on will be pleased to see that we are almost at the end. What have we llearned?

i. For small changes (up to say 20%) in velocity the conventional $p \propto Q^2$ is perfectly adequate.

ii. For larger changes e.g. 10 : 1 speed reductions the convention is far from the truth.

iii. Don't rely on the loss coefficients from ASHRAE or CIBSE guides. These often are based on tests with water where the flow is usually turbulent. For many coefficients, the original tests cannot be found, but are based on 'experience'.

iv. More experimental work is needed. The only acceptable loss coefficients are given in Eurovent 2/10 which are at least reasonable for the range of Reynolds Numbers encountered in Ventilation and Air Conditioning Systems.

v. Perhaps AMCA can persuade ASHRAE to conduct a similar experimental programme. CIBSE is now modifying its Guide, but still has a long way to go.

vi. AMCA 200-95 needs considerable modification.

A system resistance curve is likely to be of the form shown in Figure 5, although for most HVAC systems the flow at which instability occurs is very close to zero flow For mine ventilation, where the size of roadways can be considerable and the Reynolds Number is higher, this shape of system resistance curve has been recognized for at least 50 years. E.g. Ventilation Engineering by MacFarlane. It can be shown how the formula has been tailored to fit the facts by reducing the index of v velocity from 2 down to 1.9, 1.8 or even less. (See AMCA 200 Air Systems and earlier IHVE formulae).

The actual value of the index will depend on the range of velocities and sizes for which it is calculated. It will also depend on how much of the system has true laminar flow e.g. through filters.

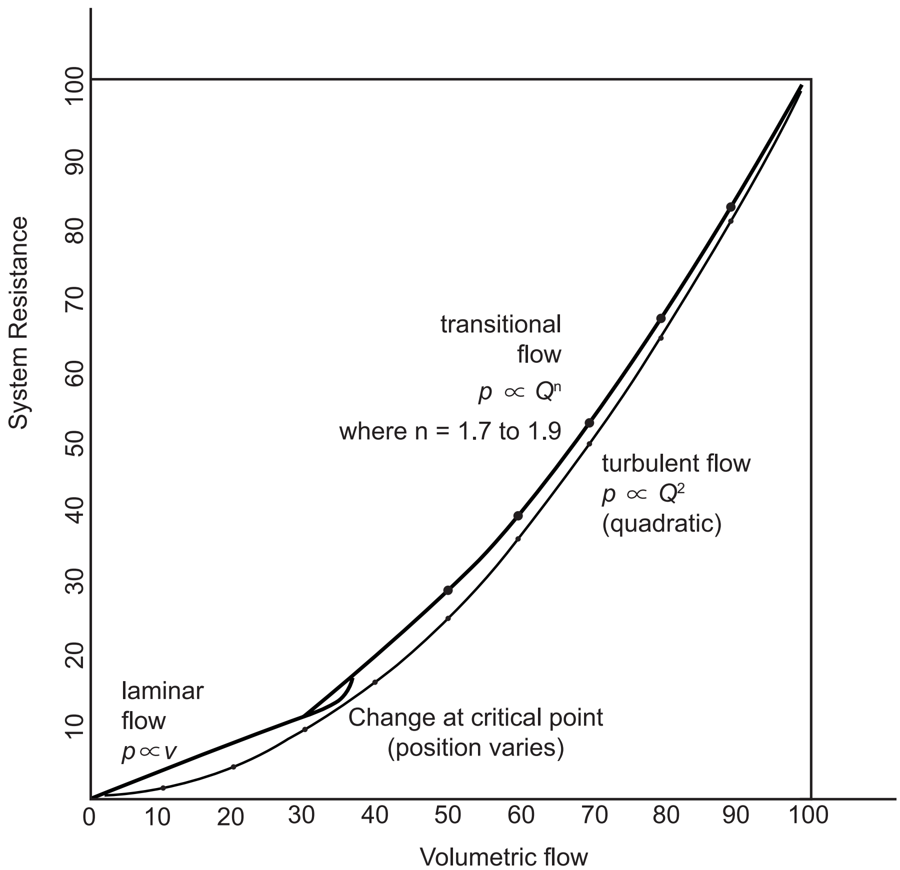

Figure 5 : A 'real' system resistance curve (typical only)

Vigilant readers of this document may detect some cynicism in that it hardly seems worth the struggle to reach the truth, if there is any!

Better to go back to basics. In this computer age, it should be possible to develop a programme to give the correct *f* for the velocity, diameter and roughness. Whether the effort is applauded, however, may still be debatable.

Norman Bolton at the National Engineering Laboratory, East Kilbride, Scotland was responsible for a programme of work, which measured the resistance of supposedly identical ducts and fittings from three different manufacturers. The variation in pressure loss p_s was enormous, thus proving that quality is everything. It also suggests that so-called balancing of systems is not enough. To use "management speak", a full system audit should be carried out and the results fed back into the company design database. Some aspects of ductwork design are rarely mentioned in textbooks and reference often has to be made to such databases.

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